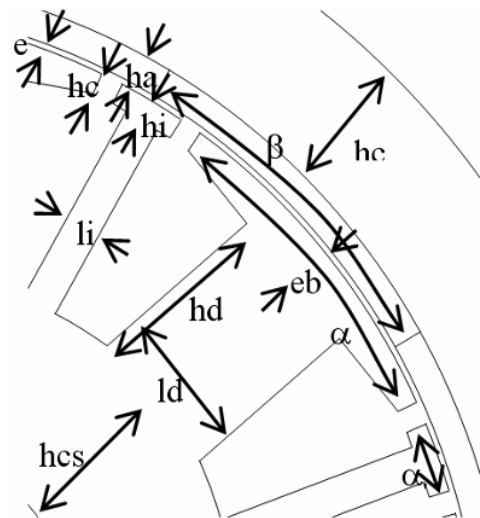
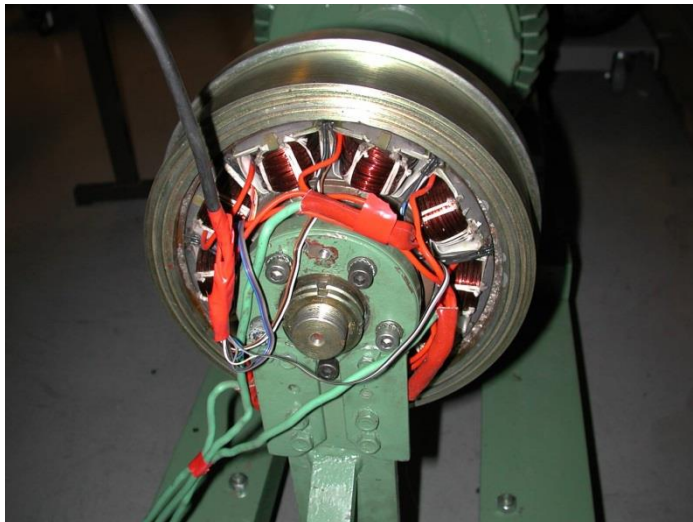


# Brushless DC Wheel Motor Benchmark

## Analytical Model & Continuous Variables

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### Motivations

Many analytical test functions are available in the literature to compare optimization algorithms. They exhibit some interesting features such as:

- explicit equations
- fast to compute
- known minimum
- scaled design variables and objectives
- no constraint

As algorithms' performances are changing a lot depending on the optimization problem and the model, the benchmark proposed here is intended to be representative of design (pre-sizing) problems in electrical engineering and more precisely in electromagnetic devices.

This benchmark exhibits other interesting features such as:

- multi-physics
- implicit equations
- highly constrained
- badly scaled design variables, objectives, and constraints
- multimodal, i.e. multiple minima

Links to fully detailed materials for the understanding and use of this benchmark are provided.

## Analytical Model

The analytical model is composed of 78 non-linear equations. The electric, magnetic, and thermal phenomena are taken into account.

The equations are distributed as follow:

- 25 for the electric and magnetic phenomena,
- 3 for the thermal one,
- 15 for the geometrical properties,
- 21 for the material properties and the configuration,
- 3 for the specifications, and
- 11 for the design variables.

This model has been presented at the Optimization and Inverse Problem in Electromagnetism (OIPE) Conference in 2004 and published in The International Journal for Computation and Mathematics in Electrical and Electronic Engineering (COMPEL), ISSN: 0332-1649, Title: Analytical model for the optimal design of a brushless DC wheel motor, Authors: S. Brisset and P. Brochet, Year: 2005, Volume: 24, Issue: 3, Page: 829-848.

The equations are detailed and explained [here](#).

The equations without comments are given [here](#).

The equations can be computed using the [Mathcad](#) file or the [Matlab](#) function.

In order to compute objective functions and constraints:

```
x = [ 1.8 ; 0.75 ; 1.6 ; 189e-3 ; 3e6 ]; % Bd(T), Be(T), Bcs(T), Ds(m), J(A/m²)
[f,g] = brushless_DC_wheel_motor_function(x,false); % 1 objective, 6 constraints
[f,g] = brushless_DC_wheel_motor_function(x,true); % 2 objectives, 5 constraints
```

where:

$$f = \begin{bmatrix} M_{tot} \\ -\eta \end{bmatrix} \quad (bi-objective) \quad \text{or} \quad f = -\eta \quad (mono-objective)$$

$$g = \begin{bmatrix} 76mm - D_{int} \\ D_{ext} - 340mm \\ -discr \\ 125A - I_{max} \\ T_a - 120^\circ C \end{bmatrix} \quad (bi-objective) \quad \text{or} \quad g = \begin{bmatrix} 76mm - D_{int} \\ D_{ext} - 340mm \\ -discr \\ 125A - I_{max} \\ T_a - 120^\circ C \\ M_{tot} - 15 \end{bmatrix} \quad (mono-objective)$$

## Optimization Problems

The aim is to have a motor with the best efficiency while respecting some technical constraints. Only five continuous and bounded design variables are kept:

$$150\text{mm} \leq D_s \leq 330\text{mm} \quad 0.5T \leq B_e \leq 0.76T \quad 0.6T \leq B_{cs} \leq 1.6T$$

$$2\text{ A/mm}^2 \leq \delta \leq 5\text{ A/mm}^2 \quad 0.9T \leq B_d \leq 1.8T$$

### Mono-objective problem

$$\begin{array}{ll} \min f & \\ \text{s.t. } g \leq 0 & \end{array} \quad \text{that is} \quad \begin{array}{ll} \max \eta & \\ \text{s.t. } M_{tot} \leq 15\text{ kg} & D_{ext} \leq 340\text{ mm} \quad D_{int} \geq 76\text{ mm} \\ I_{max} \geq 125\text{ A} & T_a \leq 120^\circ\text{C} \quad discr \geq 0 \end{array}$$

### Bi-objective problem

The second objective is to reduce the mass. The constraint on the mass is then removed to have a widespread Pareto front.

$$\begin{array}{ll} \min f & \\ \text{s.t. } g \leq 0 & \end{array} \quad \text{that is} \quad \begin{array}{ll} \max \eta & \min M_{tot} \\ \text{s.t. } D_{int} \geq 76\text{ mm} & D_{ext} \leq 340\text{ mm} \quad discr \geq 0 \\ I_{max} \geq 125\text{ A} & T_a \leq 120^\circ\text{C} \end{array}$$

## Optimization Results

### Mono-objective problem

The known global optimum is:

$B_d$	<b>1.8 T</b>	$B_e$	0.64813 T	$B_{cs}$	0.89591 T
$D_s$	201.239 mm	$\delta$	2.04370 A/mm <sup>2</sup>		
$\eta$	0.95318	$M_{tot}$	<b>15 kg</b>	$T_a$	95.346 °C
$D_{int}$	<b>76 mm</b>	$D_{ext}$	238.9 mm	$discr$	0.023599
$I_{max}$	<b>125 A</b>				

One design variable is at the upper bound and three constraints are active.

This solution is found by SQP in fmincon (Matlab Optimization Toolbox) modified with some techniques as multi-start, and scaling of variables, objective and constraints. The results are given for 1000 starting points with uniform sampling over the design space. The convergence rate is 68% and the average number of evaluations is 187. All the results of this algorithm are in the Matlab .mat file [here](#). The inputs and outputs of the model are in the order given in the text file.

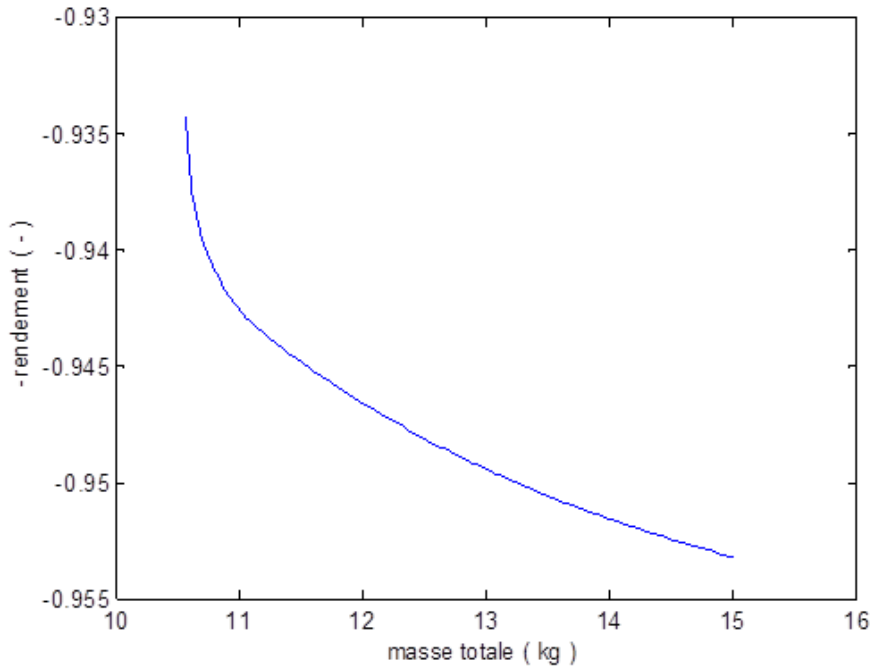
A Matlab function to run fmincon with multi-start is given [here](#). Its gives a solution slightly worst and the convergence rate is less than 10%.

The command line is below and the input argument is the number of starting points (runs):

```
[xbest, fbest, convergence]=run_fmincon_on_brushless_DC_wheel_motor_benchmark(1000)
```

## Bi-objective problem

A reference Pareto front with 100 points is given:



This solution is found by SQP in Matlab `fmincon` modified with some techniques as multi-start, scaling of variables, objective and constraints, and epsilon-constraint transformation for multi-objective problem. The results are given for 10 starting points with uniform sampling over the design space for each of the 100 solutions of the Pareto-set, resulting in 1000 mono-objective optimizations. All the results of this algorithm are in the Matlab `.mat` file [here](#).

In order to draw the Pareto front, the script is below (assuming the `.mat` file is in Matlab current folder):

```
load('bi_objective_results.mat')
plot(bi_objective_results.graph(1,:),bi_objective_results.graph(2,:))
xlabel(bi_objective_results.axes(1))
ylabel(bi_objective_results.axes(2))
```

## Contact

For any question or comment, please contact:

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*I would be glad to see the results of your algorithms on this benchmark!*