Safety Transformer Benchmark

Part II - Analytical Model & Discrete or Mixed Variables

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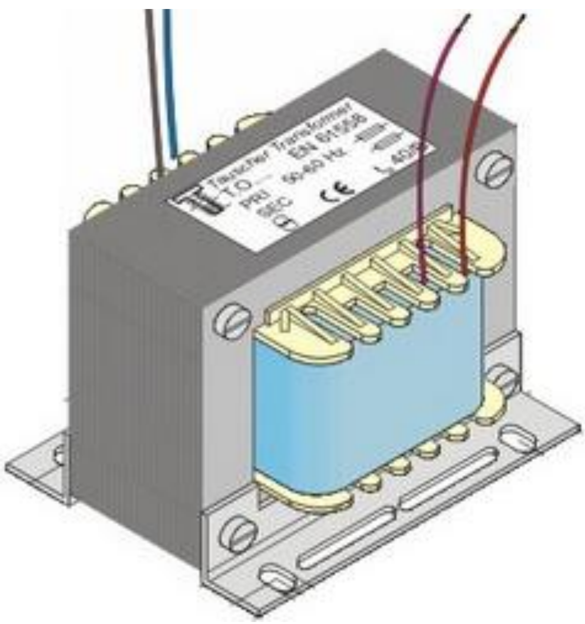
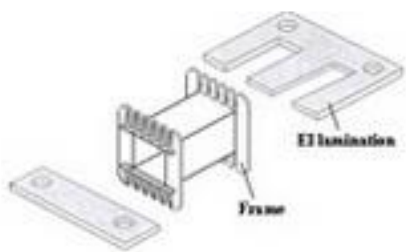




Motivations

The design of electromagnetic devices is often presented in terms of problem with continuous parameters. However, these problems are in the second part of the design process and often limited to the fine tuning of some parameters corresponding to the structure selected in the first part (working structure). There is a lack of decision tools for the choice of the structure and materials (embodiment design). In this stage, the parameters are mainly discrete and non-classable. Moreover, the production in very small series practiced by some small and medium firms must be necessarily supported by standards. It is thus a question of choosing among a great but finished number of solutions rather than to optimize some dimensions finely.

The optimization problem of the safety isolating transformer with discrete variables is detailed. An analytical model is used to compute the objectives and constraints. This model accepts continuous parameters as inputs and can thus be used with relaxation techniques.

The proposed benchmark may be used to compare combinatorial optimization algorithms, such as branch-and-bound (BB), genetic algorithms (GA), simulated annealing (SA), and other meta-heuristics.

Links to fully detailed materials for the understanding and use of this benchmark are provided.

Analytical Model

The physical phenomena within the transformer are thermal, electric and magnetic. They are expressed in equations that are ranked using specific algorithms. The assumptions for the analytical models are uniform distribution of induction in the iron core and no voltage drop due to the magnetizing current. The magnetic field in coils is vertical.

This model leads to an implicit system of 8 multi-physical equations and other equations solved sequentially. To address the multi-physical coupling, two multidisciplinary formulations are used. The multidisciplinary feasible (MDF) formulation ensures the consistency of the model and the non-linear implicit system is solved by using the fixed-point loop. As a consequence, all physics are solved several times for each model evaluation. In the individual feasible (IDF) formulation, the model is not consistent. To ensure the consistency, two additional equality constraints are used with two additional variables that link the physics. The computing time of the model is reduced as all physics are solved one time.

The model and optimization problem have been presented at the International Conference on the Computation of Electromagnetic Fields (COMPUMAG) in June 2007 and are available in the proceedings with reference: TRAN Tuan-Vu, BRISSET Stéphane, BROCHET Pascal, “A Benchmark for Multi-Objective, Multi-Level and Combinatorial Optimizations of a Safety Isolating Transformer”, COMPUMAG 2007, Aachen, Germany, 06/2007.

The equations are detailed and explained here.
The equations are given here.
The equations can be computed using the Mathcad file or the Matlab function.

In order to compute objective functions and constraints for MDF formulation:

```
x = [ 30 ; 640 ; 0.32e-6 ; 2.9e-6 ]; % abcd ; n1 ; S1(m²) ; S2(m²)
[f,g] = safety_transformer_function_abcd(x,false,false); % mono-objective
[f,g] = safety_transformer_function_abcd(x,true,false); % bi-objective
```

where:

$$f = \begin{bmatrix} M_{tot} \\ -\eta \end{bmatrix} \quad (bi-objective) \quad or \quad f = M_{tot} \quad (mono-objective)$$

$$g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ residue - 10^{-6} \end{bmatrix} \quad (bi-objective) \quad or \quad g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ residue - 10^{-6} \\ 0.8 - \eta \end{bmatrix} \quad (mono-objective)$$

In order to compute objective functions and constraints for IDF formulation:

```
x = [ 30 ; 640 ; 0.32e-6 ; 2.9e-6 ; 100 ; 1 ];
% abcd ; n1 ; S1(m²) ; S2(m²) ; Tcond_IDF(°C) ; DV2_IDF(V)
[f,g,h] = safety_transformer_function_abcd(x,false,true); % mono-objective
[f,g,h] = safety_transformer_function_abcd(x,true,true); % bi-objective
```

where:

$$f \quad unchanged$$

$$g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \end{bmatrix} \quad (bi-objective) \quad or \quad g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ 0.8 - \eta \end{bmatrix} \quad (mono-objective)$$

$$h = \begin{bmatrix} \dfrac{T_{cond}}{T_{cond\_IDF}} - 1 \\ \dfrac{\Delta V_2}{\Delta V_{2\_IDF}} - 1 \end{bmatrix} \quad (bi-objective \quad and \quad mono-objective)$$

Optimization Problems

The problem contains 7 discrete design variables. There are three parameters (a, b, c) for the shape of the lamination, one for the frame (d), two for the section of enameled wires (S_1, S_2), and one for the number of primary turn n_1. There are 24 types of lamination from catalogue r.bourgois®, 62 combinations between the laminations EI and the frames from catalogue isolectra-martin®, and 62 types of enameled wires from invex®. The number of primary turn n1 is integer but only 1001 values are allowed, leading to 246,078,000 possible combinations. All the feasible values for a, b, c, d, and S_1, S_2 are given.

There are 7 inequality constraints applied on this problem. The copper and iron temperatures T_{cond}, T_{iron} respectively are less than 120°C and 100°C. The efficiency η is greater than 80%. The magnetizing current I_{10}/I_1 and drop voltage DV_2/V_2 are less than 10%. Finally, the filling factor of both coils f_1, f_2 is lower than 1.

The objective function is to minimize the total mass M_{tot} of iron and copper materials.

The optimization problem depends on the multidisciplinary formulation. Two additional continuous design variables and two additional equality constraints are introduced in the optimizations problems with IDF formulation.

Some stochastic algorithms may have difficulty to deal with equality constraints. If so, prefer MDF formulation.

MDF formulation

The aim is to have a motor with the best efficiency while respecting some technical constraints. The fixed-point loop stops when the residue is small enough or the fixed-point loop diverges. Therefore, a constraint on the residue is added. Only 7 discrete design variables are kept for MDF formulation:

$\{a,b,d,c\} \in EI$ (62 configurations)
$n_1 \in \{200,\cdots,1200\}$ (1001 values)
$S_1 \in W \quad S_2 \in W$ (63 values each)

Mono-objective problem

$$\begin{array}{ll} \min & f \\ \text{s.t.} & g \le 0 \end{array} \quad \text{that is} \quad \begin{array}{llllll} \min & M_{tot} & & & \\ \text{s.t.} & T_{cond} \le 120°C & T_{iron} \le 100°C & \dfrac{I_{10}}{I_1} \le 0.1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 \\ & f_1 \le 1 & f_2 \le 1 & \eta \ge 0.8 & residue < 10^{-6} \end{array}$$

Bi-objective problem

The second objective is to maximize the efficiency. The constraint on the efficiency is then removed to have a widespread Pareto front.

$$\begin{array}{ll} \min & f \\ \text{s.t.} & g \le 0 \end{array} \quad \text{that is} \quad \begin{array}{llllll} \min & M_{tot} \quad \max \quad \eta & & & \\ \text{s.t.} & T_{cond} \le 120°C & T_{iron} \le 100°C & \dfrac{I_{10}}{I_1} \le 0.1 & residue < 10^{-6} \\ & f_1 \le 1 & f_2 \le 1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 & \end{array}$$

IDF formulation

Two additional continuous and bounded design variables are introduced with IDF formulation:

$\{a,b,d,c\} \in EI$ (62 configurations)
$n_1 \in \{200,\cdots,1200\}$ (1001 values)
$S_1 \in W \quad S_2 \in W$ (62 values each)
$40°C \le T_{cond\_IDF} \le 400°C \quad 0.1V \le \Delta V_{2\_IDF} \le 24V$

Mono-objective problem

$$\begin{array}{ll} \min & f \\ \text{s.t.} & g \le 0 \\ & h = 0 \end{array} \quad \text{that is} \quad \begin{array}{lllll} \min & M_{tot} & & \\ \text{s.t.} & T_{cond} \le 120°C & T_{iron} \le 100°C & f_1 \le 1 \\ & \dfrac{I_{10}}{I_1} \le 0.1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 & f_2 \le 1 \\ & \eta \ge 0.8 & T_{cond\_IDF} = T_{cond} & \Delta V_{2\_IDF} = \Delta V_2 \end{array}$$

Bi-objective problem

The second objective is to maximize the efficiency. The constraint on the efficiency is then removed to have a widespread Pareto front.

$$\min \; f \qquad \text{s.t.} \; \begin{array}{l} g \le 0 \\ h = 0 \end{array}$$

that is

$$\min \; M_{tot} \quad \max \; \eta$$

$$\text{s.t.} \quad \begin{array}{lll} T_{cond} \le 120°C & T_{iron} \le 100°C & f_1 \le 1 \\ \dfrac{I_{10}}{I_1} \le 0.1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 & f_2 \le 1 \\ \Delta V_{2\_IDF} = \Delta V_2 & T_{cond\_IDF} = T_{cond} & \end{array}$$

Optimization Results

The results are the same whatever the multidisciplinary formulation is.

Mono-objective problem

The known global optimum is:

$abcd$	36	n_1	610	S_1	0.2827mm²	S_2	2.27mm²
ΔV_{2_IDF}	1.97V	T_{cond_IDF}	109. 2°C	IDF formulation additional design variables			
M_{tot}	2.5937kg	η	0.8759	T_{cond}	109. 2°C	T_{iron}	99.58°C
f_1	0.7097	f_2	0.6434	$\dfrac{I_{10}}{I_1}$	0.0998	$\dfrac{\Delta V_2}{V_{20}}$	0.0821
$\dfrac{T_{cond}}{T_{cond\_IDF}}$	1	$\dfrac{\Delta V_2}{\Delta V_{2\_IDF}}$	1	IDF formulation additional equality constraints			

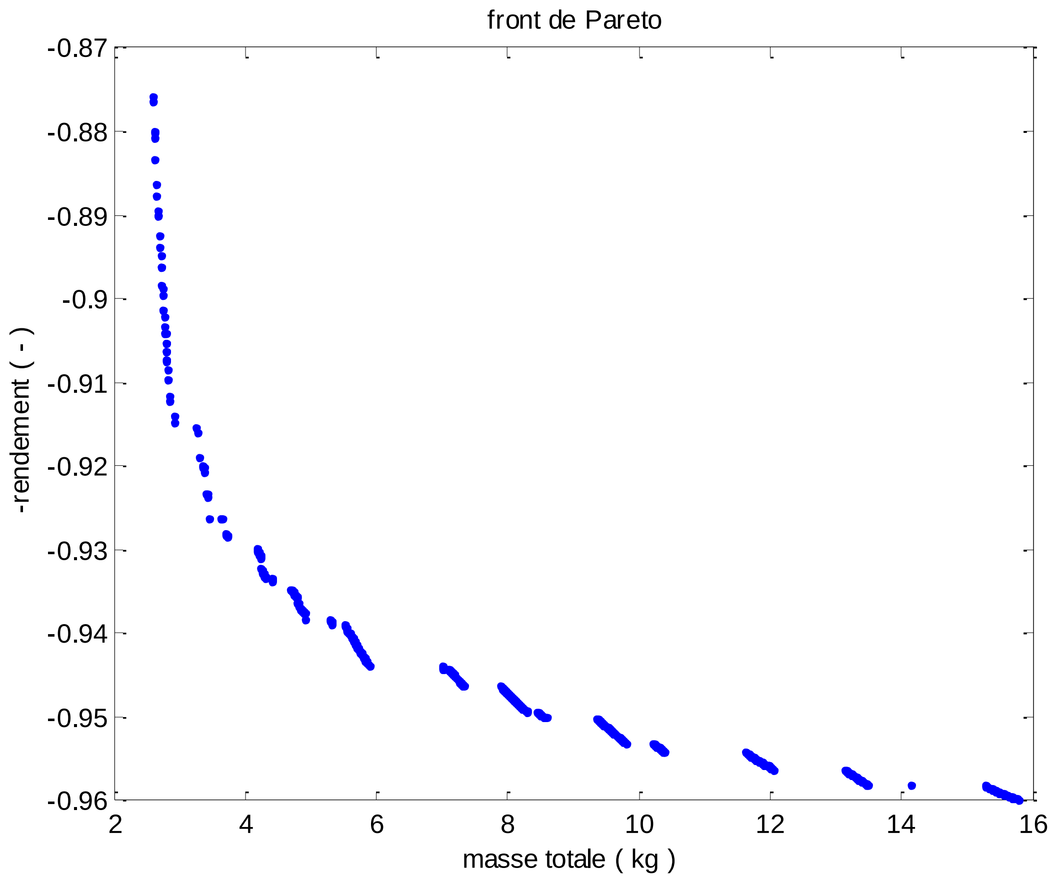
Two constraints are almost active. The values of a, b, c, and d are those of the 36th input of EI_set.txt file:

a	18mm	b	54mm	c	18mm	d	33.5mm

This solution is found by Branch and Bound algorithm with relaxation method for bounding by using SQP in fmincon (Matlab Optimization Toolbox) modified with some techniques as multi-start, and scaling of variables, objective and constraints. The results are given for 10 starting points with uniform sampling over the design space. The total number of evaluations is 284757 (MDF) or 1363208 (IDF) and the time is 585 s (MDF) or 2228 s (IDF) on a Xeon with 8 cores. All the results of this algorithm are in the Matlab .mat file here. The values of a, b, c, and d in the bestInput field should be ignored since they depend on the value of abcd that is the input number in EI_set table. The inputs and outputs of the model are in the order given in the text file.

Bi-objective problem

A reference Pareto front is given:



This solution is found by Branch and Bound algorithm with bounding made by Pareto dominance and using SQP in Matlab fmincon modified with some techniques as multi-start, scaling of variables, objective and constraints, and epsilon-constraint transformation for multi-objective problem. The results are given for 10 starting points with uniform sampling over the design. 749 Pareto solutions are found during this global search that requires 47719 seconds and 7457588 evaluations. All the results of this algorithm are in the Matlab .mat file here.

In order to draw the Pareto front, the script is below (assuming the .mat file is in Matlab current folder):

```
load('BB_results_multi_MDF.mat')
plot(BB_results_multi_MDF.graph(1,:),BB_results_multi_MDF.graph(2,:))
xlabel(BB_results_multi_MDF.axes(1))
ylabel(BB_results_multi_MDF.axes(2))
```

Contact

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I would be glad to see the results of your algorithms on this benchmark!