Safety Transformer Benchmark

Part III – Finite Element Model & Continuous Variables

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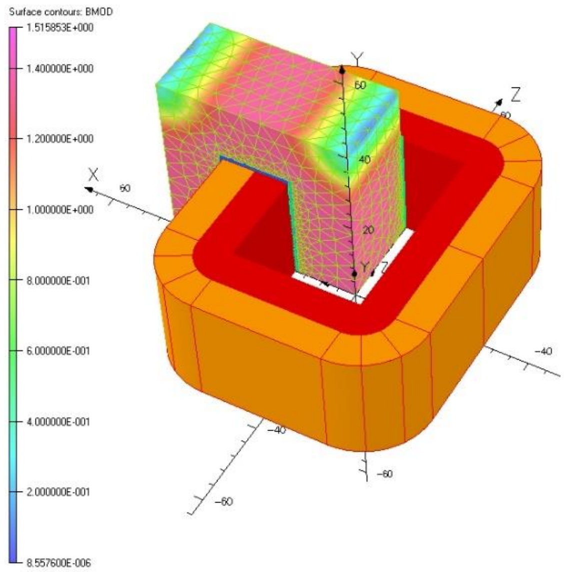
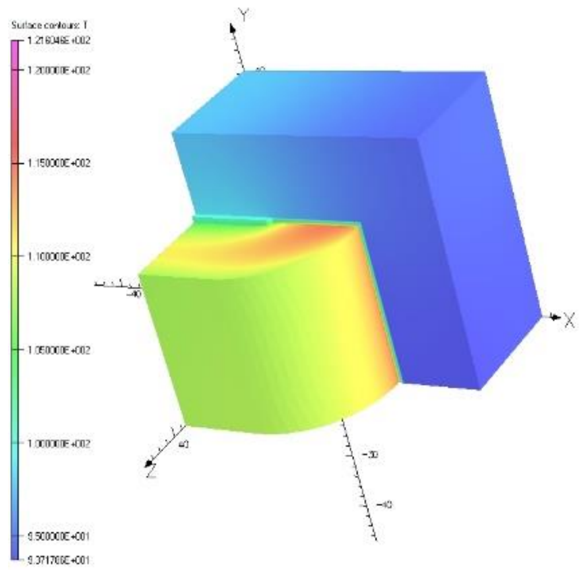




Motivations

Many test functions are available in the literature to compare optimization algorithms. They exhibit some interesting features such as:

- explicit equations
- fast to compute
- known minimum
- scaled design variables and objectives
- no constraint

As algorithms' performances are changing a lot depending on the optimization problem and the model, the benchmark proposed here is intended to be representative of design (pre-sizing) problems in electrical engineering and more precisely in electromagnetic devices.

This benchmark exhibits other interesting challenges such as:

- multidisciplinary: electric, magnetic, and thermal
- long computing time
- numerical noise due to re-meshing
- highly constrained
- badly scaled design variables, objectives, and constraints
- multimodal, i.e. multiple minima

The optimization problem can be solved with:

- optimization algorithm directly connected to the finite element (fine) model,
- meta-model based strategies, and
- multi-level techniques such as space-mapping with the availability of an analytical (coarse) model.

Links to fully detailed materials for the understanding and use of this benchmark are provided.

Finite Element (fine) Model

The physical phenomena within the transformer are thermal, electric and magnetic. Thermal and magnetic phenomena are both modeled by using 3D FEA. Since the transformer is symmetric, the simulation by 3D finite element is made on the eighth of transformer. The primary and secondary windings are both wound on the central core piece. Due to the low voltage, the primary is the inner coil that is wound around the insulation frame. There are about 43,000 nodes and 290,000 edges in the model.

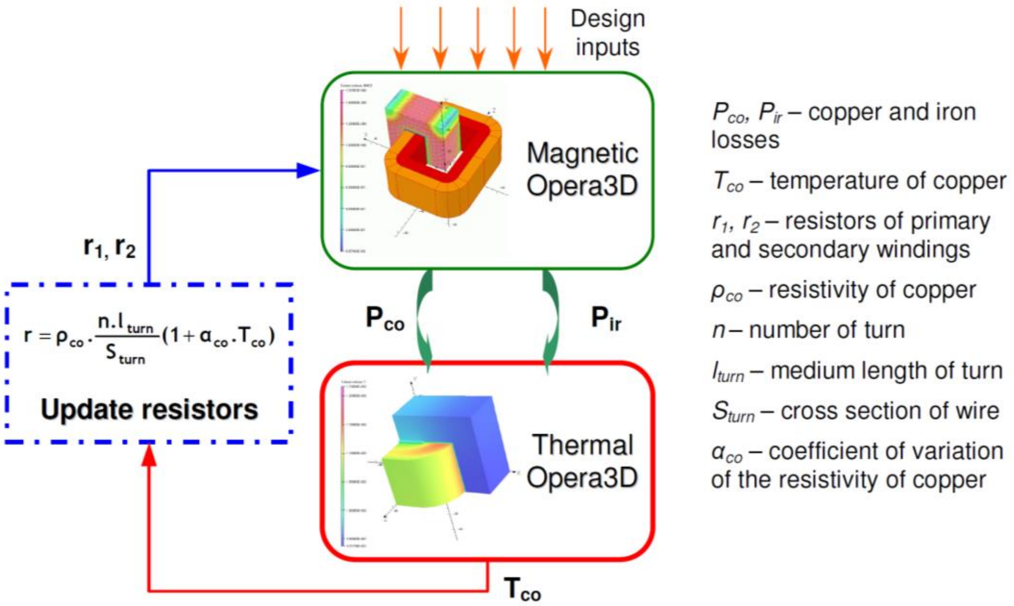
Left part of the figure on the first page shows mesh in the magnetic core, the flux-density, and the primary and secondary windings that create flux in the gap between the coils (leakage flux). For the electromagnetic modeling, all magnetic and electric quantities are assumed sinusoidal. Full-load and no-load simulations are used to compute all the characteristics. The iron loss is computed with Steinmetz formula and the leakage inductances are calculated with the magnetic coenergy.

In the thermal modeling, some assumptions are considered:

- the insulator between the core and the coils is in perfect contact with both parts
- there is no thermal contact between the exterior coil and the magnetic circuit
- there is no thermal exchange with the air trapped between the coils and the iron
- there is no convection on the upper and lower sides of the coil
- all surfaces have the same convection coefficient

The temperatures observed (right part of the figure on the first page) inside iron and core are not uniform and it appears that the copper surrounded by the magnetic circuit is hottest what corresponds to physics.

With multidisciplinary feasible (MDF) formulation, a magneto-thermal coupling is considered. The copper and iron losses are computed with the magnetic AC solver and introduced as heat sources in the thermal static solver. The copper temperature is used to compute the coils resistors introduced in the magnetic solver and this fixed-point loop continues until convergence of temperatures. The computational time of 3D FEA is about 10 minutes.



In the individual feasible (IDF) formulation, the model is not consistent. To ensure the consistency, two additional equality constraints are used with two additional variables that link the physics. The computing time of the model is reduced as all physics are solved one time.

Lumped-Mass (coarse) Model

Because 3D-FEA is very expensive in computation time, a lumped-mass "coarse" model may be used in order to speed-up the optimization process. It is available here.

Optimization Problems

The optimization problem depends on the multidisciplinary formulation. Two additional design variables and two additional equality constraints are introduced in the optimizations problems with IDF formulation.

Some stochastic algorithms may have difficulty to deal with equality constraints. If so, prefer MDF formulation.

MDF formulation

The aim is to have a motor with the best efficiency while respecting some technical constraints. The fixed-point loop stops when the residue is small enough or the fixed-point loop diverges. Therefore, a constraint on the residue is added. Only 7 continuous and bounded design variables are kept for MDF formulation:

$$6mm \leq a \leq 30mm \qquad 14mm \leq b \leq 95mm \qquad 12mm \leq c \leq 40mm$$
$$20mm \leq d \leq 80mm \qquad 200 \leq n_1 \leq 1200 \qquad 0.15mm^2 \leq S_1 \leq 19mm^2$$
$$0.15mm^2 \leq S_2 \leq 19mm^2$$

Single-objective problem

$$\min f \quad \text{s.t. } g \leq 0$$

that is

$$\min M_{tot}$$
$$\text{s.t.} \quad T_{cond} \leq 120°C \quad f_1 \leq 1 \quad \frac{I_{10}}{I_1} \leq 0.1 \quad \frac{\Delta V_2}{V_{20}} \leq 0.1$$
$$T_{iron} \leq 100°C \quad f_2 \leq 1 \quad \eta \geq 0.8 \quad residue \leq 10^{-4}$$

Bi-objective problem

The second objective is to maximize the efficiency. The constraint on the efficiency is then removed to have a widespread Pareto front.

$$\min f \quad \text{s.t.} \quad g \le 0$$

that is

$$\min \begin{bmatrix} M_{tot} \\ -\eta \end{bmatrix}$$

$$\text{s.t.} \quad T_{cond} \le 120°C \quad T_{iron} \le 100°C \quad f_1 \le 1 \quad f_2 \le 1 \quad \frac{I_{10}}{I_1} \le 0.1 \quad \frac{\Delta V_2}{V_{20}} \le 0.1 \quad residue \le 10^{-4}$$

IDF formulation

Two additional continuous and bounded design variables are introduced with IDF formulation:

$$6mm \le a \le 30mm \quad 14mm \le b \le 95mm \quad 12mm \le c \le 40mm$$
$$20mm \le d \le 80mm \quad 200 \le n_1 \le 1200 \quad 0.15mm^2 \le S_1 \le 19mm^2$$
$$0.15mm^2 \le S_2 \le 19mm^2 \quad 40°C \le T_{cond\_IDF} \le 400°C \quad 0.1V \le \Delta V_{2\_IDF} \le 24V$$

Single-objective problem

$$\min f \quad \text{s.t.} \quad g \le 0, \quad h = 0$$

that is

$$\min M_{tot}$$

$$\text{s.t.} \quad T_{cond} \le 120°C \quad T_{iron} \le 100°C \quad T_{cond} = T_{cond\_IDF} \quad f_1 \le 1 \quad f_2 \le 1 \quad \Delta V_2 = \Delta V_{2\_IDF} \quad \frac{I_{10}}{I_1} \le 0.1 \quad \frac{\Delta V_2}{V_{20}} \le 0.1 \quad \eta \ge 0.8$$

Bi-objective problem

The second objective is to maximize the efficiency. The constraint on the efficiency is then removed to have a widespread Pareto front.

$$\min f \quad \text{s.t.} \quad g \le 0, \quad h = 0$$

that is

$$\min \begin{bmatrix} M_{tot} \\ -\eta \end{bmatrix}$$

$$\text{s.t.} \quad T_{cond} \le 120°C \quad T_{iron} \le 100°C \quad T_{cond} = T_{cond\_IDF} \quad f_1 \le 1 \quad f_2 \le 1 \quad \Delta V_2 = \Delta V_{2\_IDF} \quad \frac{I_{10}}{I_1} \le 0.1 \quad \frac{\Delta V_2}{V_{20}} \le 0.1$$

Materials

The models and optimization problems have been presented at the International Conference on the Computation of Electromagnetic Fields (COMPUMAG) in June 2007 and are available in the proceedings with reference: TRAN Tuan-Vu, BRISSET Stéphane, BROCHET Pascal, “A Benchmark for Multi-Objective, Multi-Level and Combinatorial Optimizations of a Safety Isolating Transformer”, COMPUMAG 2007, Aachen, Germany, 06/2007.

The models are detailed and explained here (in French).
The 3D finite element (FE) model can be computed using the Matlab function.
Matlab Instrument Control Toolbox is required to run this function.

In order to compute objective function and constraints for MDF formulation:

```
X = [ 12e-3 50e-3 17e-3 45e-3 640 0.32e-6 2.9e-6 ; …
      13e-3 51e-3 19e-3 39e-3 640 0.32e-6 2.9e-6 ; …
      14e-3 49e-3 15e-3 43e-3 640 0.32e-6 2.9e-6 ];
% a(m) b(m) c(m) d(m) n1 S1(m²) S2(m²) each row is one evaluation
idf = false; % individual feasible (idf) formulation?
biobj = false; % bi-objective?
verbose = true; % turn it to false to avoid display
host = '193.48.25.161'; % server ip address
port = 1025; % server port
[f,g] = safety_transformer_FE_function(X,biobj,idf,verbose,host,port);
```

where:

$$f = \begin{bmatrix} M_{tot} \\ -\eta \end{bmatrix} \text{ (bi-objective) or } f = M_{tot} \text{ (mono-objective)}$$

$$g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \frac{I_{10}}{I_1} - 0.1 \\ \frac{\Delta V_2}{V_2} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ residue - 10^{-4} \end{bmatrix} \text{ (bi-objective) or } g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \frac{I_{10}}{I_1} - 0.1 \\ \frac{\Delta V_2}{V_2} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ residue - 10^{-4} \\ 0.8 - \eta \end{bmatrix} \text{ (mono-objective)}$$

In order to compute objective function and constraints for IDF formulation:

```
X = [ 12e-3 50e-3 17e-3 45e-3 640 0.32e-6 2.9e-6 100 1 ; …
      13e-3 51e-3 19e-3 39e-3 640 0.32e-6 2.9e-6 100 1 ; …
      14e-3 49e-3 15e-3 43e-3 640 0.32e-6 2.9e-6 100 1 ];
% a(m) b(m) c(m) d(m) n1 S1(m²) S2(m²) Tcond_IDF(°C) DV2_IDF(V)
idf = true; % individual feasible (idf) formulation?
[f,g,h] = safety_transformer_FE_function(X,biobj,idf,verbose,host,port);
```

where:

$$f = \begin{bmatrix} M_{tot} \\ -\eta \end{bmatrix} \text{ (bi-objective) or } f = M_{tot} \text{ (mono-objective)}$$

$$g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \frac{I_{10}}{I_1} - 0.1 \\ \frac{\Delta V_2}{V_2} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \end{bmatrix} \text{ (bi-objective) or } g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \frac{I_{10}}{I_1} - 0.1 \\ \frac{\Delta V_2}{V_2} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ 0.8 - \eta \end{bmatrix} \text{ (mono-objective)}$$

$$h = \begin{bmatrix} \frac{T_{cond}}{T_{cond\_IDF}} - 1 \\ \frac{\Delta V_2}{\Delta V_{2\_IDF}} - 1 \end{bmatrix} \text{ (bi-objective and mono-objective)}$$

Optimization results for single objective problem

IDF formulation

The best global optimum found so far is:

a	13.7mm	b	53.5mm	c	17. 2mm	d	38.6mm
n_1	677.9269	S_1	0.33905mm²	S_2	3.0142mm²		
ΔV_{2_IDF}	1.8707V	T_{cond_IDF}	119.5612°C	IDF formulation additional design variables			
M_{tot}	2.3731kg	η	0.8920	T_{cond}	119.5631°C	T_{iron}	100.0071°C
f_1	1.000000	f_2	1.000000	$\frac{I_{10}}{I_1}$	0.0999	$\frac{\Delta V_2}{V_{20}}$	0.0780
$\frac{T_{cond}}{T_{cond\_IDF}}$	1.0000	$\frac{\Delta V_2}{\Delta V_{2\_IDF}}$	1.0000	IDF formulation additional equality constraints			

All constraints are fulfilled within a tolerance of 1e-4. Five inequality constraints are active.

This solution is found by SQP in fmincon (Matlab Optimization Toolbox) modified with some techniques as multi-start, and scaling of variables, objective and constraints. The results are given for 100 starting points with uniform sampling over the design space. The convergence rate is 1% only what means that a better solution may exist. The total number of evaluations is 35163 and requires 91 hours of computing time on 10 cores.

MDF formulation

Because of the inner-loop used with MDF formulation, the computing time of the model is about 8 times greater than with IDF formulation.

The best global optimum found so far is:

a	13.9mm	b	48.7mm	c	16. 1mm	d	42.7mm
n_1	611.1676	S_1	0.3218mm²	S_2	2.873mm²		
M_{tot}	2.3737kg	η	0.8938	T_{cond}	119.9277°C	T_{iron}	99.9855°C
f_1	0.9999	f_2	0.9999	$\frac{I_{10}}{I_1}$	0.0998	$\frac{\Delta V_2}{V_{20}}$	0.0736

All constraints are fulfilled within a tolerance of 1e-4. Five inequality constraints are active.

This solution is found by SQP in fmincon (Matlab Optimization Toolbox) modified with some techniques as multi-start, and scaling of variables, objective and constraints. The results are given for 100 starting points with uniform sampling over the design space. The convergence rate is 1% only what means that a better solution may exist. The total number of evaluations is 23190 and requires 454 hours of computing time on 10 cores.

Contact

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I would be glad to see the results of your algorithms on this benchmark!